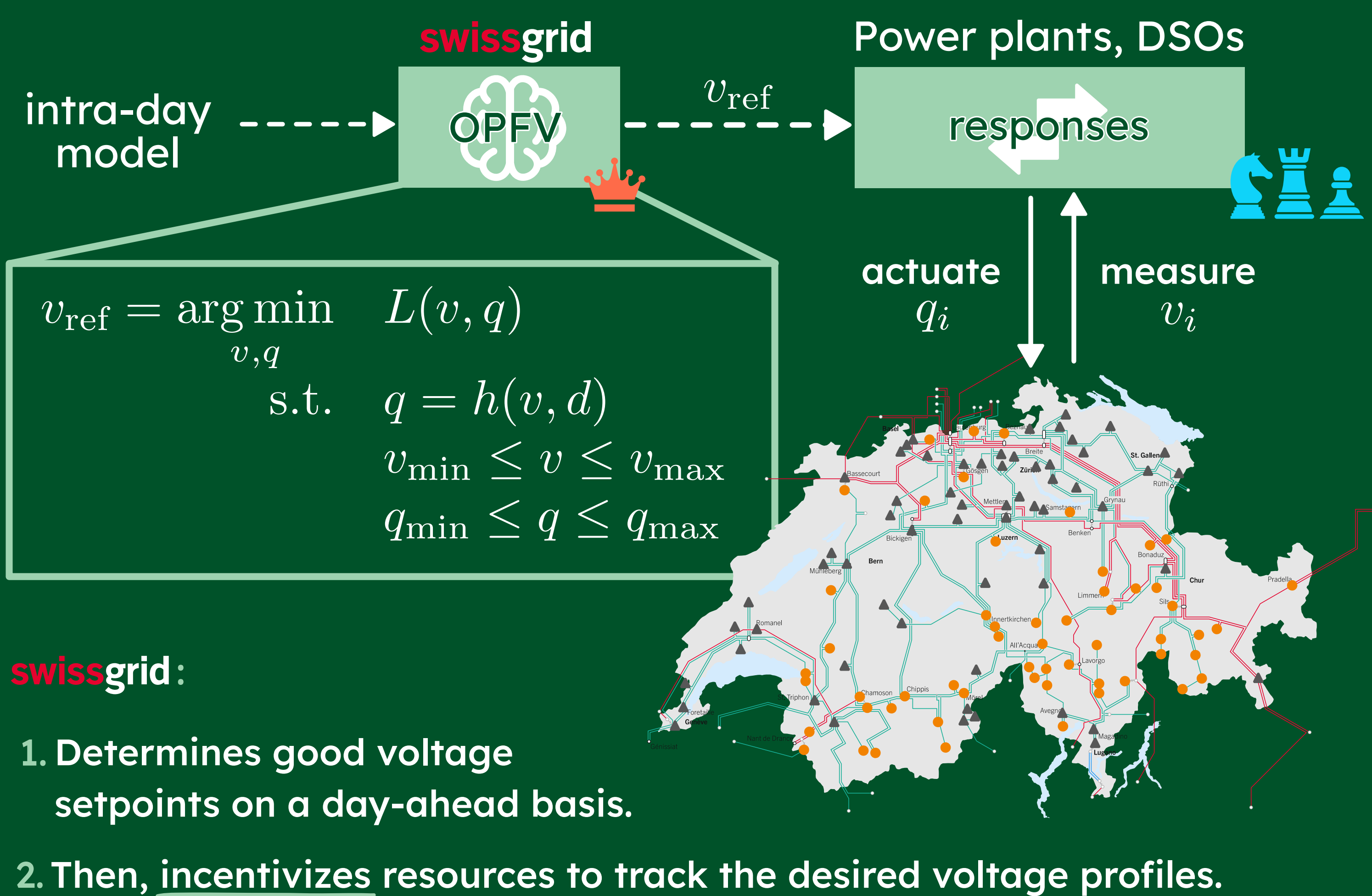


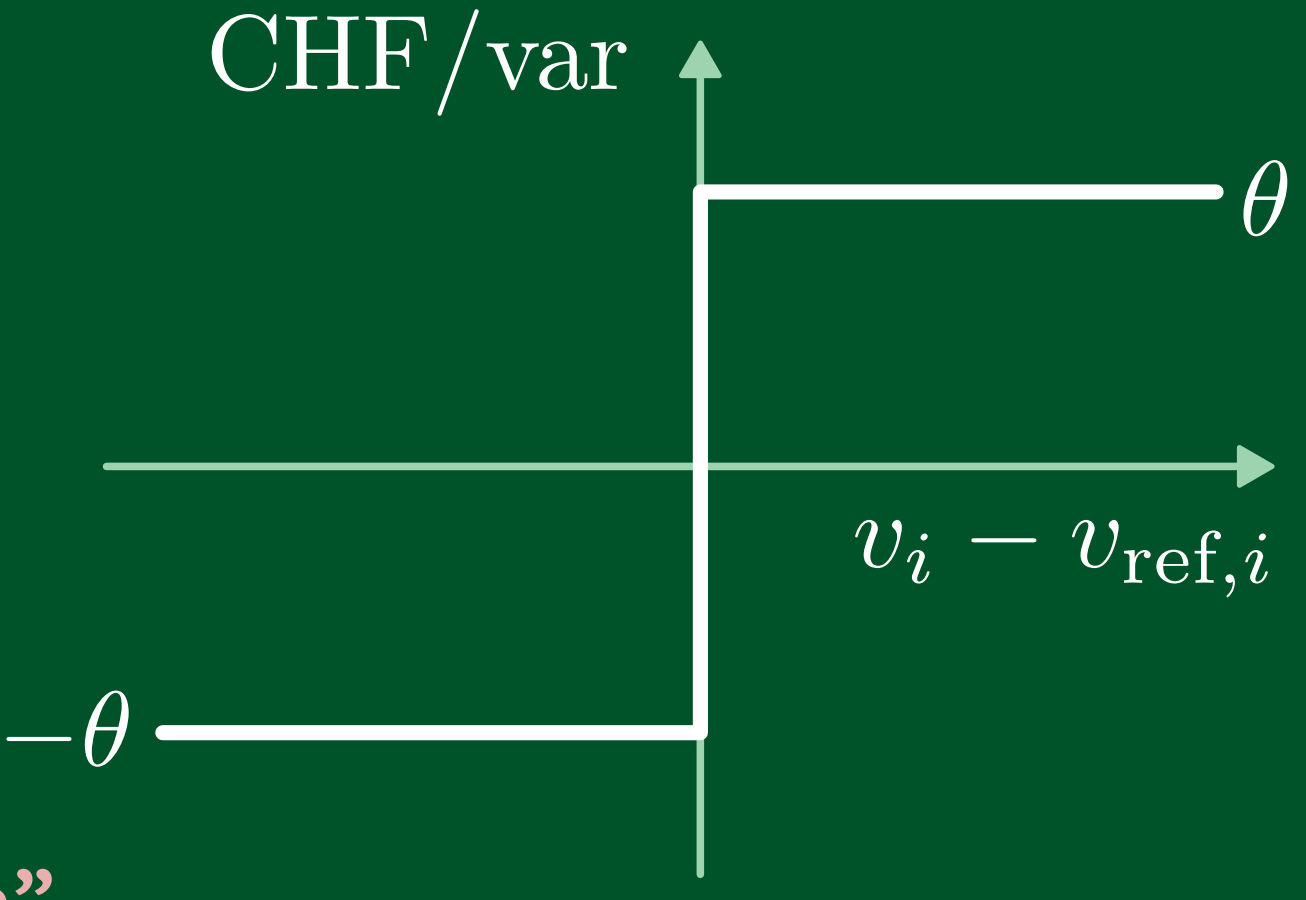
Procuring voltage support

Made in Switzerland



Incentive payments:

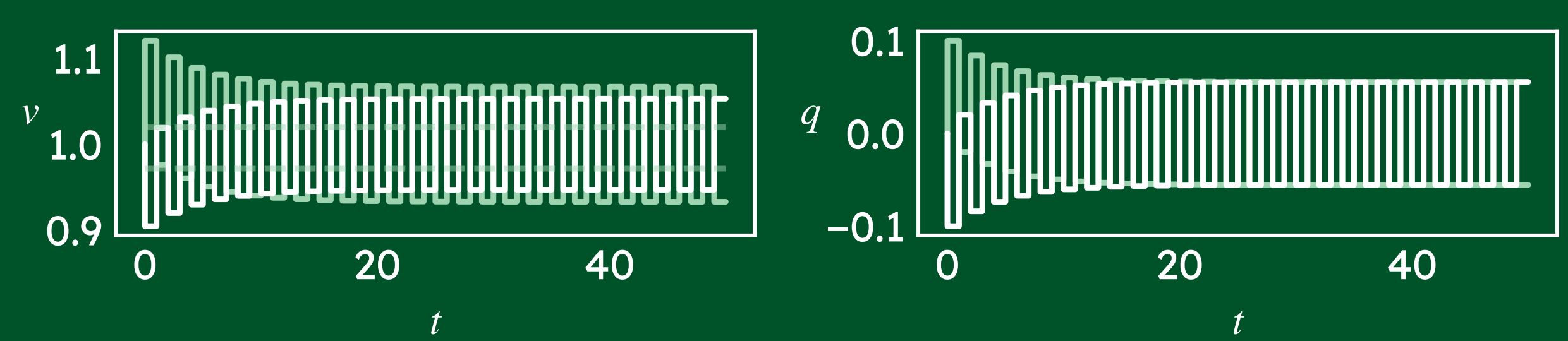
$$p(q_i, v_i; \theta) \approx \theta q_i \operatorname{sign}(v_i - v_{\text{ref},i})$$



- proportional to $|q|$
- positive (reward) if “conform”
- negative (penalty) if “non-conform”

Should incentivize tracking of reference voltage

- ! **Inefficient:** Incentive mechanism has not been designed to optimize operational costs.
- ! **Unsafe:** Discontinuous incentive function promotes destabilizing responses.

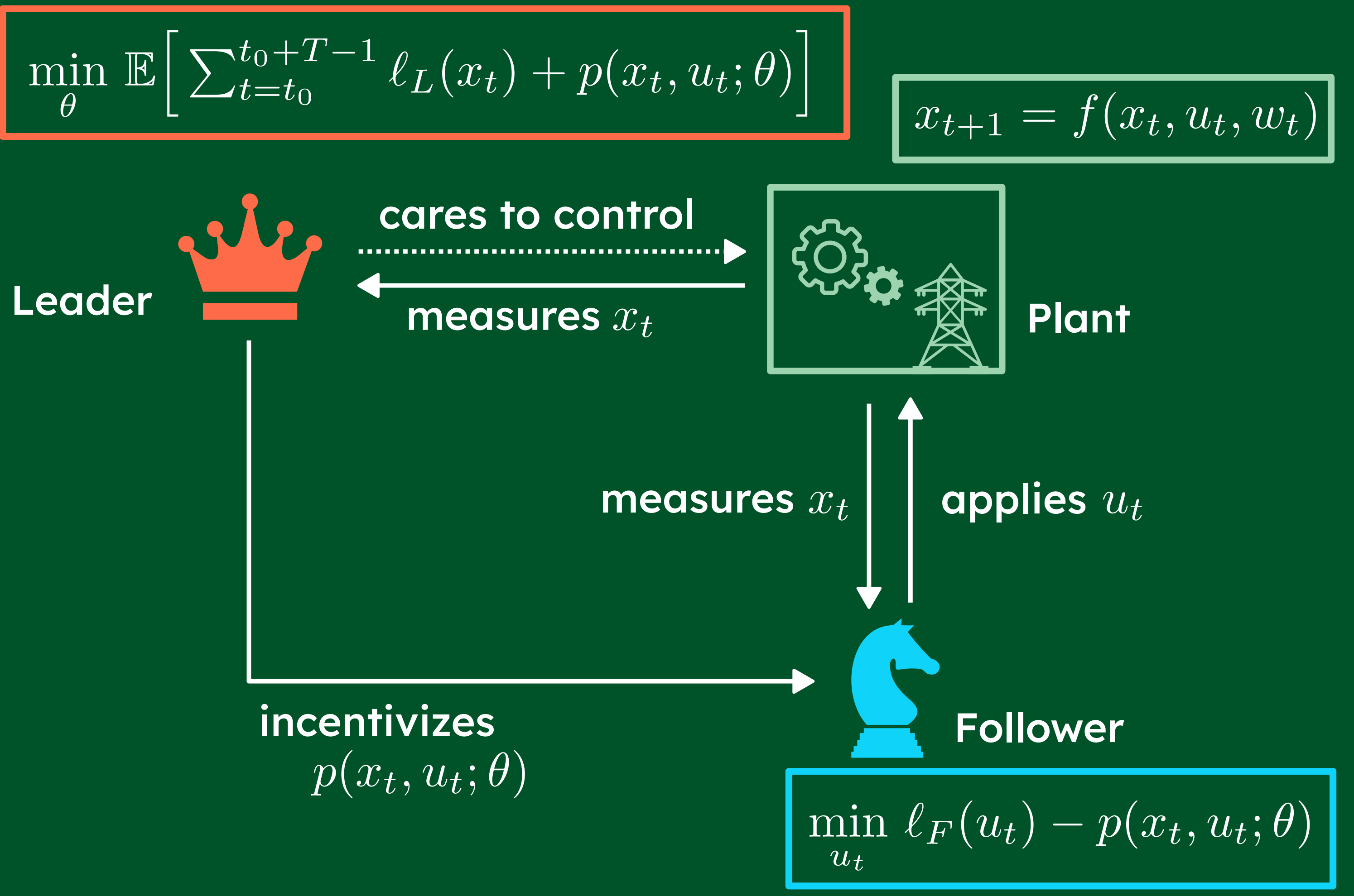


How much for your services?

Procuring control services

? What if a leader seeks to control a plant, but relies on self-interested followers to provide the control inputs?

💡 We model this problem as a dynamic Stackelberg game with an open-loop leader and a closed-loop follower.



A warm-up:
Linear-quadratic Gaussian with bilinear incentives

$$\begin{aligned} & \min_{\theta} \mathbb{E} \left[\sum_{t=t_0}^{t_0+T-1} x_t^2 + \theta x_t u_t \right] \\ & \text{s.t.} \begin{cases} \min_{u_t} c u_t^2 - \theta x_t u_t \\ x_{t+1} = a x_t + u_t + w_t \end{cases} \end{aligned} \quad p(x_t, u_t; \theta)$$

$$\begin{cases} x_0 = 0 \\ w_t \sim \mathcal{N}(0, \sigma^2) \end{cases}$$

We can find an explicit solution for this simplified problem!

$$\theta^* = \left(1 + \frac{\theta^2}{2c} \right) \frac{\sigma^2}{1 - \hat{a}^2} \left(T - \frac{\hat{a}^2(1 - \hat{a}^{2T})}{1 - \hat{a}^2} \right)$$

$$\hat{a} := a + b/2c$$

💡 The optimal leader decision has a global minimum and converges in all three problem parameters.

